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UNSTEADY HEAT TRANSFER FROM A SINGLE SPHERE IN STOKES FLOW

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Abstract—An analysis is made of the transient heat ransfer from a solid spherical particle which is suddenly introduced into a flowing fluid of different temperature. A case of very high heat conductance of the particle is considered and the velocity field around the sphere is assumed to be steady and of the Stokesian type. The problem is solved by the finite-difference method in the range of parameters $1 \leqslant Pe \leqslant 10\,000$, $0 \leqslant (\rho c)_{21} \leqslant 2$. It is found that at sufficiently large times ($\tau \to \infty$), the asymptotic (or fully developed) regime of heat transfer is approached. At $(\rho c)_{21} \geqslant 0.2$, the asymptotic values of the Nusselt number appear to be considerably less than the corresponding steady-state values for the case of constant sphere temperature. Under some conditions, values of $Nu < 2$ are obtained while local Nusselt numbers become negative in the vicinity of the rear stagnation point of the sphere.

NOMENCLATURE

a	radius of spherical particle
c	specific heat
K_1	heat conductance of the particle
M	number of mesh points in the θ-direction
N	number of mesh points in the r-direction
Nu	instantaneous Nusselt number defined by equation (12)
Nu_{st}	steady-state value of Nusselt number
Nu^*	asymptotic value of Nusselt number
Nu_θ	local Nusselt number defined by equation (11)
Pe	Peclet number, $2ua/\alpha_2$
Pr	Prandtl number, ν_2/α_2
r	dimensionless radial distance in spherical coordinate system
Re	Reynolds number, $2ua/\nu_2$
t	time
t_r	characteristic time of stabilization of thermal field near the sphere
t_h	characteristic time of stabilization of velocity field near the sphere
t_c	characteristic time of cooling (heating) of the particle
T	temperature
T_s	temperature of the sphere
T_0	initial temperature of the sphere
T_∞	temperature in the flow far from the sphere
u	free-stream velocity
V_r, V_θ	components of dimensionless velocity in spherical coordinate system
x	transformed r-coordinate, $r = \exp(x)$
Δx	step size in radial direction
Z	dimensionless temperature, $(T-T_\infty)/(T_0-T_\infty)$
$Z^*(r,\theta)$	normalized dimensionless temperature in asymptotic regime.

Greek symbols

α	thermal diffusivity
β	coefficient defined by equation (16)
β_1	first positive root of equation (23)
$\Delta\tau$	time-step size
$\Delta\theta$	angular step size
θ	polar angle in spherical coordinate system
ν	kinematic viscosity
ρ	density
$(\rho c)_{21}$	ratio of volume heat capacity of the fluid to that of the sphere, $(\rho_2 c_2)/(\rho_1 c_1)$
τ	dimensionless time, $\alpha_2 t/a^2$.

Subscripts

1	sphere
2	fluid.

Superscript

*	asymptotic quantities (at $\tau \to \infty$).

1. INTRODUCTION

THE CLASSICAL problem of heat transfer from a single spherical particle moving in a surrounding fluid at low Reynolds number has been the subject of numerous investigations during the past two decades (see, for example, the review of Clift *et al.* [1]). Most of the theoretical work is concerned with the case of steady heat transfer, in which the temperature of the particle is maintained constant and both the velocity and thermal fields around the particle are completely stable. However, in many technological applications (such as liquid-fluidized bed reactors or spray columns) the heat transfer between the particles and the continuous phase is always essentially unsteady. Nevertheless, in practice, calculation of the heating or cooling of particles in a surrounding flow is often based on the quasi-steady approach. The heat transfer coefficients are assumed to be independent of time and equal to their steady-state value at a fixed sphere temperature and at time-averaged hydrodynamical conditions. The overall Nusselt numbers are evaluated using various semi-empirical correlations which for the force-convective heat transfer are usually expressed in the form

$$Nu_{st} = 2 + F(Re, Pr). \quad (1)$$

Here, $F(Re, Pr) \to 0$ as $Re \to 0$. The constant '2' is the theoretical minimum of the steady Nusselt number for the sphere in a stagnant medium.

It is important to note that the quasi-steady approach is valid only if the processes of stabilization (relaxation) of hydrodynamical and thermal fields near the surface of the sphere are of much shorter duration than the characteristic time of cooling (or heating)† of the particle

$$t_h \ll t_c, \quad (2)$$

$$t_r \ll t_c. \quad (3)$$

Conditions (2) and (3) are far from satisfied in many cases, especially for solid–liquid and liquid–liquid systems and therefore the analysis of unsteady heat transfer seems to be very important. In general, this problem is very complicated. In the present paper a simplified treatment of the problem will be given. In particular, our model postulates that the velocity field near the spherical particle is steady during the cooling process, i.e. inequality (2) is satisfied.

The problem was analysed in a similar formulation by a number of authors. Bentwich *et al.* [2] studied the time-dependent temperature distribution near the sphere which is suddenly introduced into the flow of a fluid at a different temperature. The temperature of the sphere, T_s, is assumed to be constant, and the flow around the particle steady and potential. The problem was solved by the numerical method in the range of the intermediate Peclet number ($1 < Pe < 1000$).

Konopliv and Sparrow [4, 5] examined transient heat transfer problems for a sphere inserted in a uniform velocity field at small Peclet numbers ($Pe \leqslant 1$). In another work by the same authors [6], the case of Stokesian flow around a sphere at high Peclet numbers ($Pe \gg 1$) was analysed with the use of the model of thin thermal boundary layers. The thermal conductance of the particle was assumed to be very high such that the sphere temperature was spatially uniform at any instant of time.

Brauer [7] presented numerical results for the unsteady mass transfer problem from a spherical particle in creeping flow. It was found that in some cases, the direction of mass transfer was reversed in the vicinity of the rear stagnation point, although no physical explanation was given for this phenomenon.

In the present work the problem statement is similar to that of Konopliv and Sparrow [6], but the range of the parameters studied is different. In particular, we are interested in basically unsteady conditions, for which inequality (3) is not valid.

† Mathematical descriptions of the problems of cooling and heating of a solid body in a flow are identical. We restrict the following discussion to the cooling problem.

2. STATEMENT OF THE PROBLEM AND THE PRINCIPAL ASSUMPTIONS

Consider a solid spherical particle with radius a, having an initial temperature, T_0, which is suddenly immersed in an unbounded laminar flow of a different temperature T_∞ ($T_\infty < T_0$). Assume, that the heat conductance of the particle is so high that there are no temperature gradients within the sphere at each instant of time. As was shown by Konopliv and Sparrow [3], this assumption seems a reasonable one for metallic particles in a gaseous or liquid environment.

Assume that the flow field around the particle is axisymmetric and steady, the fluid velocity, u, is uniform far from the sphere, and the Reynolds number $Re = 2ua/\nu_2$ is small compared to unity. For such creeping flow around a sphere, the non-dimensional velocity components are expressed as

$$V_r = -\tfrac{1}{2}[2r^2 - 3r + 1/r] \cos \theta, \quad (4)$$

$$V_\theta = \tfrac{1}{4}[4r - 3/r - 1/r^3] \sin \theta. \quad (5)$$

Here, r is the dimensionless radius, and θ is the polar angle of the spherical coordinate system which is fixed at the centre of the sphere. The angle θ is measured from the frontal stagnation point on the sphere's surface.

The unsteady convective heat transfer from the particle is governed by the following dimensionless energy equation for the fluid

$$\frac{\partial Z}{\partial \tau} + \frac{Pe}{2}\left[V_r \frac{\partial Z}{\partial r} + \frac{V_\theta}{r}\frac{\partial Z}{\partial \theta}\right] = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial Z}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial Z}{\partial \theta}\right). \quad (6)$$

The initial and boundary conditions are

$$\tau = 0, \quad Z_s = 1; \quad Z = 0, \quad (7)$$

$$r = 1, \quad Z = Z_s(\tau); \qquad \frac{dZ_s}{d\tau} = \frac{3}{2}\frac{(\rho_2 c_2)}{(\rho_1 c_1)} \int_0^\pi \left(\frac{\partial Z}{\partial r}\right)_{r=1} \sin\theta \, d\theta, \quad (8)$$

$$r \to \infty, \quad Z = 0, \quad (9)$$

$$\theta = 0; \quad \pi \frac{\partial Z}{\partial \theta} = 0. \quad (10)$$

In these equations, $Z = (T - T_\infty)/(T_0 - T_\infty)$ is the dimensionless fluid temperature; Z_s is the particle temperature; $\tau = \alpha_2 t/a^2$ is the dimensionless time (Fourier number); $Pe = 2ua/\alpha_2$ is the Peclet number; ρ, c and α are the density, heat capacity and thermal diffusivity, while subscripts 1 and 2 refer to particle and surrounding fluid, respectively.

The initial condition (7) indicates that at $\tau = 0$ the local temperature of the flow near the sphere is uniform and equal to T_∞. Condition (8) is obtained by equating the overall surface heat transfer to the change of internal energy of the solid. Equation (9) shows that far from the sphere the temperature of the fluid remains undisturbed at any times τ. Condition (10) follows from the symmetry of the temperature field along the stagnation lines $\theta = 0, \pi$.

The local instantaneous Nusselt number is defined by

$$Nu_\theta = -\frac{2}{Z_s}\left(\frac{\partial Z}{\partial r}\right)_{r=1}, \tag{11}$$

while the surface-averaged value of the Nusselt number is calculated from

$$Nu = -\frac{1}{Z_s}\int_0^\pi \left(\frac{\partial Z}{\partial r}\right)_{r=1} \sin\theta \, d\theta$$

$$= -\frac{2}{3}\left(\frac{\rho_1 c_1}{\rho_2 c_2}\right)\frac{d \ln Z_s}{d\tau}. \tag{12}$$

In these definitions, the driving force for the heat transfer is taken as the difference between the instantaneous particle temperature T_s and the fluid free-stream temperature T_∞.

3. METHOD OF SOLUTION

The unsteady energy equation (6) was solved by the finite-difference method of alternating direction (ADI) in the range of Peclet numbers $1 \leqslant Pe \leqslant 10\,000$.

The conventional transform $r = \exp(x)$ is introduced for the radial coordinate. The use of a constant step size for x made it possible to obtain a denser mesh near the sphere's surface, where the temperature gradients are relatively large and, hence, the accurate difference approximation is needed.

The external boundary condition (9) is assumed to be valid at a large but finite distance r_∞ from the centre of the sphere. This distance was chosen as a function of Pe, in accordance with the recommendations suggested in the literature (refs. [10, 11] and others). For instance, at $Pe = 1$, $r_\infty \simeq 55$, while at $Pe = 1000$, $r_\infty \simeq 5$ were taken. The step size for the angular coordinate is uniform, $\Delta\theta = \pi/40 = 4.5°$. The number of mesh points in the radial direction is $N = 50 \div 61$. The time step is variable and changes from $\Delta\tau = 10^{-5}$ at the start of computations to $\Delta\tau = 0.001 \div 0.01$ at the final stage. The integral boundary condition (8) on the surface of the sphere was calculated by the explicit method. Symmetry conditions (10) were approximated with second-order accuracy.

To increase the stability of the finite-difference scheme, the 'upwind' approximation was used for the convective terms in equation (6) at $Pe > 100$ [9]. For lower values of Peclet numbers standard space-centred differencing was applied. At $Pe < 100$ the results obtained both by upwind and central differencing differ only slightly. This can be seen from Table 1, where the influence of the mesh parameters and outer boundary on the overall Nusselt numbers is also shown. These data illustrate the 'grid independence' of the numerical solution.

Table 1. Effect of mesh parameters on computation of asymptotic Nusselt number†

Pe	$(\rho c)_{21}$	r_∞	$N \times M$	Nu^*
10	0	10	26 × 21	3.244
($\Delta\tau = 0.01$)		28	51 × 41	3.244
		28‡	51 × 41	3.182
	1	10	26 × 21	0.877
		28	51 × 41	0.874
		28‡	51 × 41	0.838
1000	1	5	26 × 21	8.799
($\Delta\tau = 0.001$)		5	51 × 41	8.572
		10	51 × 41	8.620

† $\Delta\tau$ is the time step at the final stage of computations.
‡ Upwind scheme.

4. RESULTS AND DISCUSSION

In addition to the Peclet number, the other important parameter of the problem is the ratio of the volume heat capacity of the surrounding flow to that of the particle material: $(\rho c)_{21} = (\rho_2 c_2)/(\rho_1 c_1)$. For liquid–liquid and solid–liquid systems this parameter ranges within the limits of 0.3–3. In the present study, the parameter $(\rho c)_{21}$ was varied in the range 0–2. The value $(\rho c)_{21} = 0$ corresponds to the special case of constant sphere temperature (infinitely large heat capacity of the particle: $\rho_1 c_1 \to \infty$).

4.1. *The limiting case of Pe = 0*

The analytical solution for the important case of unsteady heat transfer of a highly conductive sphere in a stagnant fluid was presented by Konopliv and Sparrow [3] in the form

$$Z(r,\tau) = \frac{2\gamma}{\pi r}\int_0^\infty \zeta \exp(-\zeta^2\tau) \times \frac{\zeta \cos[(1-r)\zeta] - (1-\gamma\zeta^2)\sin[(1-r)\zeta]}{(1-\gamma\zeta^2)^2 + \zeta^2} d\zeta, \tag{13}$$

where $\gamma = (\rho_1 c_1)/3(\rho_2 c_2)$.

We applied solution (13) to calculate the temperature of the particle $Z_s = Z_s(\tau)$, and the instantaneous Nusselt numbers according to expression (11). These results are shown in Fig. 1 for different values of $(\rho c)_{21}$. The asymptotic behaviour (at $\tau \to \infty$) of the Nusselt number may be represented as

$$Nu \doteq \frac{1}{(\rho c)_{21}\tau}. \tag{14}$$

In the case of a constant sphere temperature, the Nusselt number is calculated using the well-known expression [13]

$$Nu = 2\left(1 + \frac{1}{\sqrt{\pi\tau}}\right). \tag{15}$$

In the final stage of the cooling process, the Nusselt numbers drop far below the 'classical' minimal value of

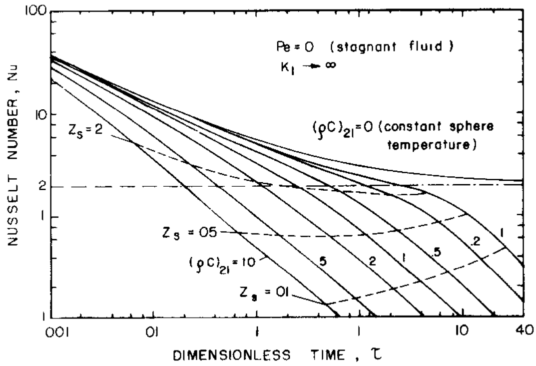

FIG. 1. Time-variation of the instantaneous Nusselt number for the sphere inserted into stagnant fluid. Dotted lines indicate the temperature of the sphere.

$Nu_{st} = 2$ for the steady-state heat transfer. To provide a qualitative explanation of the possibility, we consider a somewhat different physical situation as shown in Fig. 2. Assume that over a sufficiently long period the particle was maintained at a constant temperature T_s. At time $t = t_*$, the steady temperature distribution is established in the fluid around the particle (solid line in Fig. 2). Assume now that the source of heating is suddenly turned off, and the sphere begins to cool down. The temperature distribution near the sphere at time $(t_* + \Delta t)$ is shown by the dotted line. Far from the particle, the temperature profile at time $(t_* + \Delta t)$ is practically coincident with that at time t_*. This is because the thermal disturbance, which is initiated by the changing particle temperature, penetrates in the radial direction only on a distance $\Delta R \simeq \sqrt{(\alpha_2 \Delta t)}$ during the time interval Δt. If the thermal disturbance is propagated in the fluid at an infinite rate (quasi-steady state), the temperature distribution depicted by the solid line would be obtained at time $(t_* + \Delta t)$. Comparing such a quasi-steady profile with the actual one, we may conclude that in a given situation the transient Nusselt number must be less than the corresponding steady value: $Nu(\tau) < Nu_{st} = 2$.

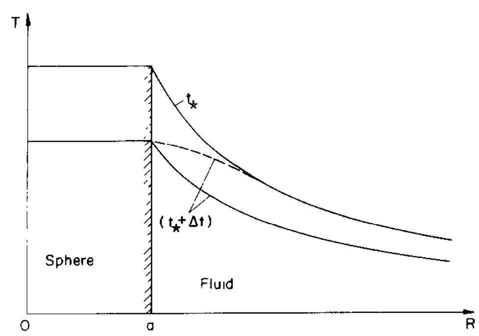

FIG. 2. Qualitative picture of temperature distribution near a cooling particle at times t_* and $(t_* + \Delta t)$. Stagnant fluid, steady initial distribution at $t = t_*$. ———, quasi-steady profile; – – – –, actual profile.

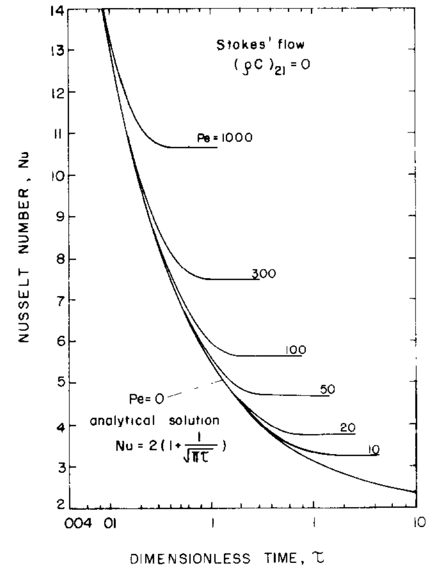

FIG. 3(a). Temporal development of Nusselt number at different Pe. Case of constant sphere temperature.

4.2. *Intermediate Peclet numbers* ($1 \leqslant Pe \leqslant 1000$)

We start our discussion with Fig. 3(a) where the transient behaviour of the average Nusselt numbers is shown for the case of constant sphere temperature and at different Pe. The instantaneous Nusselt numbers are initially very large, rapidly decreasing with time. At short times, the numerical solutions at all Peclet numbers coincide with the analytical solution (15) for a sphere in a stagnant fluid. The influence of forced convection is manifested from the times $\tau \simeq 1/Pe^{2/3}$, and at this stage the Nusselt numbers decrease asymptotically approaching the steady-state values of

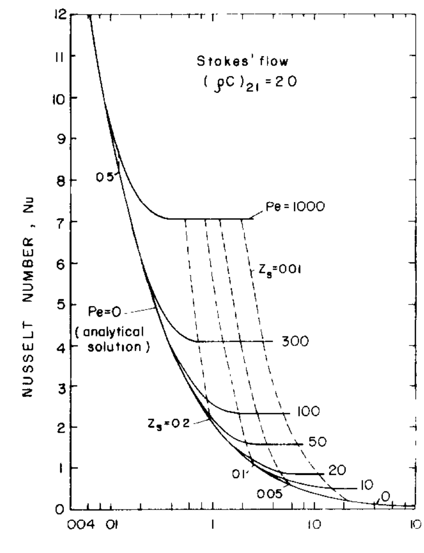

FIG. 3(b) Temporal development of Nusselt number at different Pe. Sphere temperature changing with time: $(\rho c)_{21} = 2$. Dotted lines show the values of Z_s.

Table 2. Asymptotic Nusselt numbers as a function of Pe and of $(\rho c)_{21}$†

Pe	$(\rho c)_{21}$ 0 (T_s = const.)	0.2	0.5	1.0	2.0
1	2.299	0.234	0.087	0.043	0.022
	2.30‡	(0.523)	(0.219)	(0.111)	(0.056)
10	3.244	2.292	1.461	0.873	0.476
	3.25‡	(2.369)	(1.591)	(0.993)	(0.557)
20	3.763	2.972	2.161	1.433	0.838
	—	(3.018)	(2.252)	(1.536)	(0.920)
50	4.690	4.030	3.263	2.420	1.558
	—	(4.058)	(3.324)	(2.505)	(1.643)
100	5.621	5.025	4.289	3.393	2.344
	5.62‡	(5.045)	(4.336)	(3.467)	(2.430)
300	7.497	6.974	6.285	5.358	4.108
	7.65‡	(6.980)	(6.300)	(5.383)	(4.115)
1000	10.657	10.184	9.529	8.572	7.071
	11.01‡	(10.182)	(9.528)	(8.578)	(7.098)
10000	22.043	21.633	21.037	20.094	18.387
	22.87‡	(21.610)	(20.985)	(20.006)	(18.259)

† The values predicted by the 'film' model are given in parentheses.
‡ Numerical solution of Brian and Hales [12].

Nu_{st}. These steady values are in good agreement with the numerical results of Brian and Hales [12]. Using the data of Fig. 3(a), we can now define more strictly the characteristic time of stabilization of the thermal field around the particle τ_r, as the time required for the instantaneous Nusselt number $Nu(\tau)$, to approach the steady value with some preassigned accuracy (5%, for instance). At $Pe = 0$, equation (15) yields: $\tau_r \simeq 127$, while for $Pe \geqslant 1$ our numerical results may be approximated by $\tau_r \simeq 1.5/(Nu_{st}-2)^2$. At very large Peclet numbers, the latter formula is close to the expression $\tau_r \simeq 1.6/Pe^{2/3}$, which may be obtained from the boundary-layer solution of Konopliv and Sparrow [6].

The temporal development of the Nusselt numbers in the case of time-changing sphere temperature is shown in Fig. 3(b). A considerable portion of heat is transferred in the conduction regime. As time elapses, the overall Nusselt numbers decrease up to some constant values. These asymptotic values, Nu^*, which are also listed in Table 2, are found to be significantly below the corresponding steady-state Nusselt numbers. In a number of cases, the values $Nu^* < 2$ are obtained. It may be noted that the times required to attain the asymptotic Nusselt number, τ_r, are practically independent of the parameter $(\rho c)_{21}$. The characteristic time of cooling, τ_c, may be estimated from these data as a time required for the particle temperature Z_s to reduce to 5–10% of its initial value (i.e. up to $Z_s = 0.05$–0.1). As can be observed, for the cases presented in Fig. 3(b), the characteristic times τ_r and τ_c are of the same order of magnitude.

Figure 4 shows the development in time of the local Nusselt numbers at $Pe = 300$ and $(\rho c)_{21} = 2$. Near the forward stagnation point ($\theta = 0$), where the temperature boundary layer is comparatively thin, the transient thermal process is completed more rapidly than that in other parts of the sphere.

As can be seen from Fig. 4, the local heat transfer coefficients may well become negative in the vicinity of the rear stagnation point ($\theta = 180°$). In this region, the heat is transferred not from the sphere to the surrounding flow, but in the opposite direction. A possible physical explanation for this seemingly anomalous result may be offered. Consider a fluid

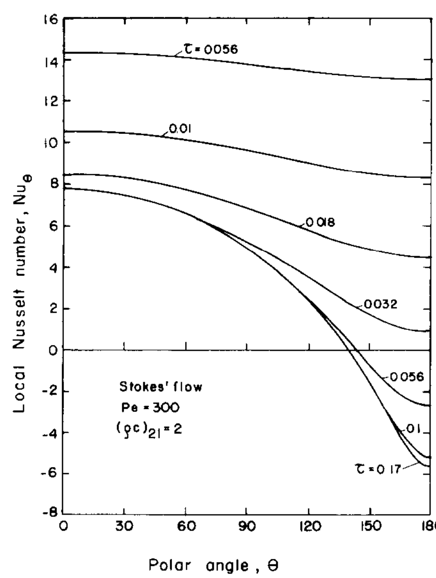

FIG. 4. Local Nusselt numbers at different times. $Pe = 300$, sphere temperature changing with time: $(\rho c)_{21} = 2$.

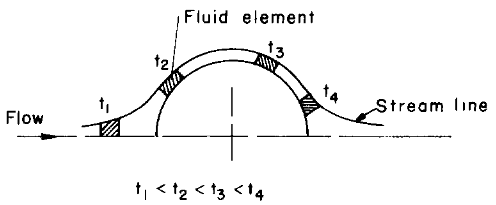

FIG. 5. Sketch of motion of a liquid element along the sphere surface.

element, which is moving in the vicinity of the sphere surface, taking in successive times $t_1, t_2, \ldots,$ the positions depicted in Fig. 5. At time $t \geqslant t_1$ the fresh cold element encounters the hot particle, and here the heat transfer is maximal. As a fluid element shifts along the surface, its mean temperature increases, and therefore the local temperature gradients (values of Nu) are reduced with the polar angle θ. Simultaneously, the temperature of the particle decreases, primarily as a result of a strong heat loss from the leading part of the sphere. At some instant of time (say, t_4), the sphere temperature may drop below the temperature of the liquid element, and here the reversal heat transfer will be observed. Obviously, such a situation is more probable at large values of the parameter $(\rho c)_{21}$ and low fluid velocity (or low Peclet numbers). Note, that in addition to the above-mentioned study of Brauer [7], similar effects are also reported by Wittke and Chao [8] for heat transfer between a single vapour-gas bubble and the surrounding liquid.

In Figs. 6 and 7 the asymptotic local Nusselt numbers, Nu_θ^*, are shown at different Pe and $(\rho c)_{21}$. The separation angle θ_s, for which $Nu_\theta^* = 0$, shifts from the rear stagnation point to the equator plane ($\theta = 90°$) with increasing values of the parameter $(\rho c)_{21}$ or reduction of Pe. In the asymptotic regime at low Peclet

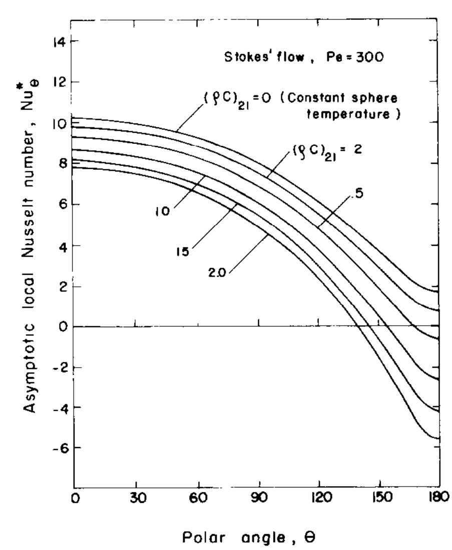

FIG. 6. Asymptotic distribution of local Nusselt numbers at $Pe = 300$ and different values of parameter $(\rho c)_{21}$.

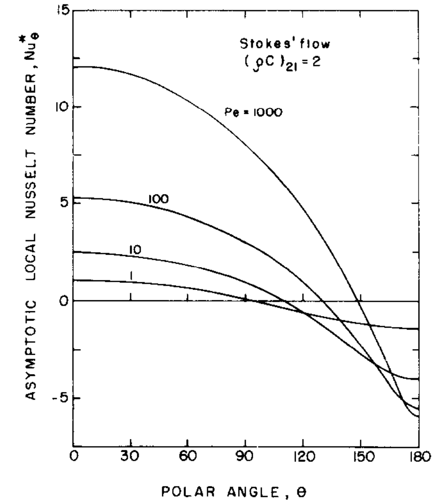

FIG. 7. Asymptotic distribution of local Nusselt number at $(\rho c)_{21} = 2$ and different Pe.

numbers, a major portion of the heat transferred from the leading part of the sphere is returned to the rear part. It must be noted, however, that at $Pe = 1$ the appearance of negative values of Nu_θ is observed only in the final stages of the cooling process, when the excess temperature of the particle $(T_s - T_\infty)$ becomes less than 1% of its initial value.

Figure 8 shows the isotherm contours in the flow past the sphere at large times, when the asymptotic distributions of Nu are attained. Here the numbers on the curves represent the values of the normalized temperature $Z = Z(r, \theta, \tau)/Z_s(\tau)$. Figure 8(a) corresponds to the case of steady heat transfer from a particle with a constant temperature at $Pe = 300$. When the sphere temperature changes with time, an 'attached thermal wake' may appear behind the sphere [Figs. 8(b) and (c)]. The temperature within the thermal wake is larger than the particle temperature. As $(\rho c)_{21}$ increases, the thermal wake lengthens and its maximal temperature grows with respect to the temperature of the sphere.

A similar 'thermal wake' has also been observed at intermediate Reynolds numbers in our preliminary study [14].

4.3. *Asymptotic behaviour (at $\tau \to \infty$)*

At very large times τ, when the overall and local Nusselt numbers approach the corresponding constant values, the temperature of the sphere decreases as an exponential function of time: $Z_s \sim \exp(-\beta\tau)$, where [see equation (12)]

$$\beta = \tfrac{3}{2}(\rho c)_{21}\, Nu^*. \qquad (16)$$

Considering equation (11), we may conclude that the temperature gradients on the sphere surface decrease in exactly the same way: $(\partial Z(r, \theta, \tau)/\partial r)_{r=1} \sim \exp(-\beta\tau)$. It becomes apparent that at large times, the temperature of the fluid in the vicinity of the particle can

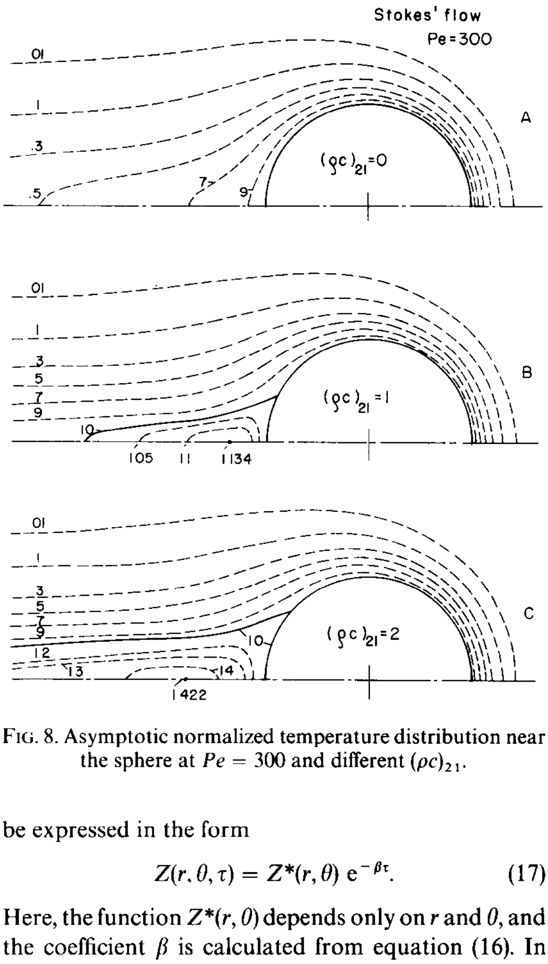

FIG. 8. Asymptotic normalized temperature distribution near the sphere at $Pe = 300$ and different $(\rho c)_{21}$.

be expressed in the form

$$Z(r,\theta,\tau) = Z^*(r,\theta)\,\mathrm{e}^{-\beta\tau}. \tag{17}$$

Here, the function $Z^*(r,\theta)$ depends only on r and θ, and the coefficient β is calculated from equation (16). In other words, the temperature profiles near the sphere appear to be self-similar with time. This asymptotic regime, which is sometimes also called the 'fully developed regime' has been discussed previously only for transient heat transfer within the bounded bodies [18].

It should be emphasized, however, that the representation (17) is not rigorously valid at sufficiently large distances from the surface of the sphere. The simplest physical analysis shows that at each point of the fluid, the temperature first increases when the thermal disturbance produced by the particle reaches this point. Thus there will always be points in the flow where the temperature increases at a given instance of time, contrary to equation (17). After reaching a maximum value, the temperature at each point will decrease with time.

Let us assume for the time being, that in the limit $\tau \to \infty$, the representation (17) is valid also at large distances from the particle. Then, substituting (17) into equation (6), we obtain that the function $Z^*(r,\theta)$ is governed by the equation

$$\frac{Pe}{2}\left[V_r\frac{\partial Z^*}{\partial r}+\frac{V_\theta}{r}\frac{\partial Z^*}{\partial \theta}\right]=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial Z^*}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Z^*}{\partial\theta}\right)+\beta Z^*. \tag{18}$$

At the sphere's surface, the function Z^* can arbitrarily be set equal to 1, so that $Z^*(r,\theta)$ will then represent the normalized temperature in the asymptotic regime. At infinity ($r \to \infty$) $Z^* \to 0$. Hence, the boundary conditions for equation (18) may be written in the form

$$r = 1, \quad Z^* = 1, \tag{19}$$

$$r \to \infty, \quad Z^* = 0. \tag{20}$$

The steady-state boundary-value problem (18)–(20) may be solved numerically, but the difficulty is that parameter β is not known initially. This parameter may be found by the 'trial-and-error' method using the addition condition

$$\beta = -\frac{3}{2}(\rho c)_{21}\int_0^\pi\left(\frac{\partial Z^*}{\partial r}\right)_{r=1}\sin\theta\,\mathrm{d}\theta, \tag{21}$$

which is obtained from equation (8) or equation (16).

This way of calculation seems to be very laborious, and therefore in the present study the inverse problem was solved. Namely, a series of positive values of β was chosen, and for each β the numerical solution of equation (18) with boundary conditions (9) and (20) was found. Next, from equation (21) the values of parameters $(\rho c)_{21}$ corresponding to given values of β, were determined.

To solve the asymptotic problem (18)–(20), a 'false transient term' $\partial Z^*/\partial\tau$, was introduced into the LHS of equation (18) and the above-described alternating direction method was used.† As an 'initial' temperature distribution, the solution obtained either in the steady-state case ($\beta = 0$), or for the previous value of β, was taken. The iterations were repeated until the convergence condition $|Nu_\theta^{(j+1)} - Nu_\theta^{(j)}| < 10^{-5}$ was reached for all θ. Here the superscript j denotes the number of the iteration.

The results of the numerical solution are presented in Fig. 9, where the overall Nusselt number is plotted as a function of the parameter β at $Pe = 300$. In Fig. 9, the asymptotic values of Nu^* obtained from the transient cooling problem at large τ, are also shown. The excellent agreement between the transient and the asymptotic solution provides indirect support for the above assumption, that at large times, the temperature in the flow around the cooling sphere decreases in accordance with equation (17).

Note that equations similar to (18)–(20) were solved by Chen and Pfeffer [15] for the problem of steady mass transfer from a sphere with a first-order homogeneous chemical reaction ($\beta < 0$) in the flowing fluid. Chambre and Young [17] studied a generative reaction (positive β) in a laminar boundary layer flow past a flat plate. They found, in particular, that the distance from the leading edge to the point where the mass transfer reversed its direction was proportional to u/β.

The asymptotic Nusselt number Nu^* may be

† The presence of the source term, βZ^*, in equation (18) has a de-stabilizing effect on numerical solution, hence the upwind scheme is preferable here.

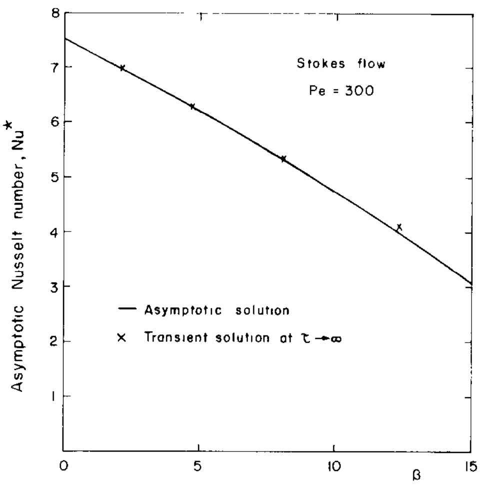

FIG. 9. Comparison of the asymptotic Nusselt numbers obtained from the transient solution with those predicted by the asymptotic model.

satisfactorily predicted by means of a simple one-dimensional 'film model' which is widely used in chemical engineering calculations [16]. According to this model, the actual convective–conductive transfer from the particle is simulated by the purely conductive transport through the stagnation liquid film of constant thickness $\delta_f = 2a/(Nu_{st}-2)$. Here, the steady-state Nusselt number Nu_{st} is assumed to be known under given hydrodynamical conditions [see equation (1), for instance].

Finally, the asymptotic Nusselt number may be expressed as

$$Nu^* = \frac{2\beta_1}{3(\rho c)_{21}}, \tag{22}$$

where β_1 is the first positive root of the equation

$$[\beta - 3(\rho c)_{21}] \tan\left(\frac{2\sqrt{\beta}}{Nu_{st}-2}\right) = 3(\rho c)_{21}\sqrt{\beta}. \tag{23}$$

In Table 2 approximate calculations using equations (22) and (23) are compared with our numerical solution for the sphere in Stokes flow. Asymptotic Nusselt numbers predicted by the film model are given in brackets. The film model presents accurate estimates of the asymptotic heat transfer rate at $Pe \geqslant 10$. At small Peclet numbers ($Pe < 10$) and large values of $(\rho c)_{21}$ the zone of 'negative' heat transfer occupies a considerable part of the sphere's surface (see Fig. 7), and here the film model leads to an overestimation for Nu^*. Overall, we can conclude that the film model provides a good first approximation for the asymptotic Nusselt number.

5. CONCLUSIONS

The unsteady heat transfer from a solid spherical particle in the Stokes flow was studied by the finite-difference method in the case of intermediate Peclet numbers and high thermal conductivity of the sphere. It is found that when the parameter $(\rho c)_{21}$ is varied in the range 0.2–2, the relaxation time of the thermal field in the vicinity of the sphere's surface, and the characteristic time of cooling (heating) of the particle are of the same order of magnitude. Under these conditions, the asymptotic regime of heat transfer (at $\tau \to \infty$) may be considerably different from the steady-state heat transfer. The asymptotic Nusselt number is significantly lower than its corresponding steady-state value. In some cases, the local Nusselt numbers at the rear part of the sphere become negative and an 'attached thermal wake' appears behind the sphere.

The asymptotic Nusselt number may be satisfactorily estimated by the use of a simple 'film' model.

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TRANSFERT THERMIQUE VARIABLE POUR UNE SPHERE UNIQUE DANS UN ECOULEMENT DE STOKES

Résumé—On analyse le transfert thermique variable pour une particule sphérique qui est introduite soudainement dans un fluide en écoulement, à température différente. On considère un cas de très haute conductance de la particule et le champ de vitesse autour de la sphère est supposée permanent et de type Stokesien. Le problème est résolu par la méthode aux différences finies pour le domaine de paramètres $1 \leqslant Pe < 10\,000$, $0 \leqslant (\rho c)_{21} \leqslant 2$. On trouve qu'à des temps suffisamment grands ($\tau \to \infty$), le régime thermique asymptotique (ou pleinement développé) est approché. A $(\rho c)_{12} \geqslant 0{,}2$, les valeurs asymptotiques du nombre de Nusselt sont considerablement moindre que les valeurs correspondantes de l'état stationnaire pour le cas de la température de sphère constante. Sous certaines conditions, des valeurs de $Nu < 2$ sont obtenues tandis que des nombres de Nusselt locaux deviennent négatifs au voisinage du point d'arrêt arrière de la sphère.

INSTATIONÄRE WÄRMEÜBERTRAGUNG VON EINER EINZELNEN KUGEL AN EINE STOKES'SCHE STRÖMUNG

Zusammenfassung—Es wird der instationäre Wärmeübergang an einer starren Kugel untersucht, die plötzlich in ein strömendes Fluid von anderer Temperatur eingebracht wird. Dabei wird der Fall sehr hoher Wärmeleitfähigkeit der Kugel betrachtet und angenommen, daß das Geschwindigkeitsfeld um die Kugel stationär und vom Stokes'schen Typ ist. Das Problem wird mit der Methode der finiten Elemente gelöst für Bereiche der Peclet-Zahl von $1 \leqslant Pe \leqslant 10\,000$ und für ρc von $0 \leqslant \rho c \leqslant 2$. Es zeigt sich, daß für genügend große Zeiten ($\tau \to \infty$) der asymptotische (oder vollständig ausgebildete) Bereich des Wärmeübergangs erreicht wird. Für $\rho c \leqslant 0{,}2$ liegen die asymptotischen Grenzwerte der Nusselt-Zahl beträchtlich unterhalb der Werte für den entsprechenden stationären Zustand bei konstanter Temperatur der Kugel. Unter bestimmten Bedingungen wird $Nu < 2$ erreicht, und die lokale Nusselt-Zahl wird in der Nähe des hinteren Staupunktes der Kugel negativ.

НЕСТАЦИОНАРНЫЙ ТЕПЛОПЕРЕНОС ОТ ЕДИНИЧНОЙ СФЕРЫ, ОБТЕКАЕМОЙ СТОКСОВЫМ ПОТОКОМ

Аннотация—Проведен анализ переходного процесса теплопереноса от твердой сферической частицы, мгновенно вводимой в поток жидкости с другой температурой. Рассматривается случай очень высокой теплопроводности частицы в предположении, что поле скорости вокруг нее имеет стационарный стоксовский характер. Задача решается методом конечных разностей в диапазоне параметров $1 \leq Pe \leq 10\,000$, $0 \leq (\rho c)_{21} \leq 2$. Найдено, что при относительно больших временах ($\tau \to \infty$) достигается асимптотический (или полностью развитый) режим теплопереноса. При $(\rho c)_{21} \geq 0{,}2$ асимптотические значения числа Нуссельта оказываются значительно меньшими, чем соответствующие стационарные значения для случая постоянной температуры сферы. При некоторых условиях получены значения $Nu < 2$, в то время как локальные числа Нуссельта принимают отрицательные значения вблизи задней критической точки сферы.